# Efficient Markets and Chaos: A Perturbation Approach

Charles G. Masi and Glendon Williams

Department of Business and Technology, Keiser University

## Author Note

Charles G. Masi https://orcid.org/0000-0003-0322-3931

The version of this paper printed in the *Journal of Strategic and International Studies* contains errors on p. 86. Specifically, the graphs for Figures 4 and 5 have been switched, and the image labeled Figure 4 was flipped both top/bottom and left/right, rendering it unreadable. Both errors have been corrected in this manuscript version.

Correspondence concerning this article should be addressed to Charles G. Masi, 27488 Bourbonniere Drive, Bonita Springs, FL 34135, United States. email: cgmasi@cgmasi.com

**Abstract**

The 87 year period from the end of the Great Depression to the start of the COVID-19 pandemic exhibited relatively stable economic growth in the United States, providing a useful laboratory for studying behavior of U.S. equities markets. This research used perturbation analysis to develop a framework for modeling the behavior of the Dow Jones Industrial Average from April of 1934 through December of 2019. The theoretical framework consisted of an exponential growth component representing growth of the underlying economy as predicted by the efficient market hypothesis (EMH) combined with a chaotic perturbation component modeled as signals from an ensemble of phase-shift oscillators operating at closely spaced frequencies. Analysis of historical data showed that during the period of interest an exponential component with amplitude coefficient of 0.258 with an exponent of 0.04876 ($R^2 = 0.9855$) plus a chaotic component consisting of an ensemble of 11,445 oscillators correctly modeled DJIA market-price activity. Normalized chaotic oscillations showed an RMS amplitude close to one third of the exponential growth component (standard deviation equal to 0.3325). The oscillations' power spectrum was a fractal following Zipf's Law with a -1.832 exponent ($R^2 = 0.6684$).

*Keywords:* efficient markets, EMH, Dow Jones Industrial Average, DJIA, gross domestic product, GDP, perturbation analysis, chaos, control theory, oscillation, investment decision, buy-sell-hold strategy, Fourier analysis

**Efficient Markets and Chaos: A Perturbation Approach**

Since Hayek (1945) suggested a mechanism by which free markets set prices for goods and services as a group consensus seeking an equilibrium price, scholars have sought a quantitative explanation for why actual market prices so often diverge from the expected equilibrium prices (Fama, 1970). This article proposes an answer to this long-standing research question using an approach based on an application of perturbation theory and tests it against historical records.

Hayek's mechanism posited that, while no single investor had enough knowledge to accurately predict a market price for a particular good or service, negotiations between buyers and sellers would collaboratively settle on an equilibrium market price that would accurately reflect its value. Subsequent scholars have endlessly debated models to explain markets' obvious and consistent departures from the equilibrium price, known as excess volatility (Bouchard, 2021; Fama, 1970; Turcaș et al., 2022; Shiller, 2003).

When, in the middle of the 20th century, Edward Lorenz (1963) revolutionized the study of non-linear dynamical systems by introducing chaos theory, economic theorists quickly began trying to apply it to explain the market-disequilibrium problem, but with so-far marginal results (Inglada-Perez, 2020; Turcaș et al., 2022).

Shiller (2003) suggested that, while EMH does appear to explain long-term trends in equities markets, the excess volatility "had its origins in human foibles and *arbitrary feedback relations* [emphasis added]" (p. 102). The study reported in this paper followed Shiller's suggestion by positng that the excess volatility observed is, indeed, the result of a feedback mechanism acting on a chaotic assemblage of individual investors who act as both buyers and sellers in equities markets (Turcaș et al., 2022). This additional volatility acts as a *perturbation* on the EMH-mediated market equilibrium operating as Hayek (1945) described (Weinberg, 2021).

The next section describes a framework that incorporates a chaotic mechanism for perturbations to the EMH model. The third section steps back to review the literature on EMH, market dynamism and chaos theory. The fourth section describes methodology to test the model against historical market data. The fifth section describes the results obtained. The final section discusses the implications of those results and suggests further research to improve the framework.

## Literature Review

This paper builds upon scholarly literature in two areas: efficiency of markets setting prices for equities and other valuable products and services, and chaos in large dynamical systems.

### Market Efficiency and Equities Price Setting

The efficient market hypothesis (EMH) has three forms (Klock, & Bacon, 2014):

- *Weak-form EMH* refers specifically to predictions based on past-price information;
- *Semi-strong form EMH* includes use of all publicly available information;
- *Strong-form EMH* includes all information, including private, company-confidential information.

The framework proposed in this paper envisions market-price movements as chaotic fluctuations around an equilibrium value determined by strong-form market efficiency (Chauhan et al., 2014; Gleick, 2008).

## Stock-Market Dynamism

Once a stock is sold to the public, it can be traded between various investors at a strike price that is agreed upon *ad hoc* between buyers and sellers in a secondary market (Hayek, 1945). When one investor decides to sell stock in a given company, it increases the supply of that stock for sale, exerting downward pressure on the strike price. Conversely, when an investor decides to buy that stock, it increases the demand, driving the strike price up. Interestingly, consummating the transaction decreases both supply and demand, and thus has no effect on the strike price. It is the intention to buy or sell the stock that affects the price. The market price is the strike price of the last transaction completed.

Successful firms grow in value over time, which is reflected in secular growth of the market price of their stocks (Fama, 1970). GDP growth reflects the overall growth of the value of successful companies that make up the national economy. Thus, EMH suggests that overall market value (measured by appropriately constituted market indices) should reflect GDP secular growth.

Of course, if all investors were assured the market price would rise, no owners would be willing to sell, no transactions could occur, and the market would collapse (Hayek, 1945). Similarly, if all investors were assured that the stock's market price would fall, owners would be anxious to sell, but nobody would be willing to buy. Again, no transactions could occur, and the market would, again, collapse. Markets therefore actually work because of the dynamic tension created by uncertainty as to whether any given stock's market price will rise or fall in the near future, making equities markets dynamical systems that move constantly.

Fama (1970) concluded that on time scales longer than a day, the EMH appeared to work. He found, however, evidence that on shorter time scales it was possible to use past-price information to obtain returns in excess of market returns, violating even weak-form efficiency. He concluded, however, that returns available on such short time scales were insufficient to cover transaction costs, upholding weak-form EMH (Klock, & Bacon, 2014). Technological improvements since 1970 have, however, drastically reduced costs for high volumes of very-short-timescale transactions, making high-frequency trading profitable (Baron et al., 2019).

**Chaotic Dynamics in Large Systems**

Such short-time predictability and long-time unpredictability is a case of *sensitive dependence on initial conditions*, which was discovered by Edward Lorentz in 1961 to be one of the hallmarks of chaos (Gleick, 2008). Since 1970, considerable work has been published applying the science of chaotic systems to markets, especially the forex market, which operates nearly identically to equities markets (Bhattacharya et al., 2017).

Chaos is a property of dynamical systems (Strogatz, 2018). To illustrate how a dynamic system can become oscillatory requires an easily understood example, such as the pitch-control system in an aircraft (Efremov et al., 1996). This system uses HIL negative-feedback control (Brogan, 1985) with two moving parts: the pilot and aircraft. In that system, the oscillation arises from a difference in the speed at which the aircraft reacts to control inputs, and the speed at which the pilot reacts to correct aircraft movements (Efremov et al., 1996). The pilot's response typically lags the aircraft's movement by a more-or-less fixed time. In such a case, there is always an oscillation frequency at which that time lag equals one half of the oscillation period (i.e., time to complete one cycle) causing the feedback's effect to shift from negative to positive as indicated in Figure 2. The aircraft's nose then bobs up and down at the oscillation frequency, giving the aircraft a porpoising motion. Should the pilot try to control the porpoising, the oscillation only grows larger because the response still lags the motion by the same amount, but the gain of the system (output amplitude divided by input amplitude) increases with pilot effort to exert control. This is called *pilot involved oscillation (PIO)*, and it is a major nuisance for all HIL feedback-control systems.

PIO relates to stock-market behavior because there is also a lag between market-price movement and any given investor's reaction to set a price based on it (Baron et al., 2019). The time lag between intention and consummation of a trade will necessarily represent half the period of some PIO-like oscillation. The fact that at any given time there are multiple investors (up to many thousands) driving market-price fluctuations at their own individual oscillation frequencies (determined by their individual execution-time lags) makes the overall market a chaotic system with many closely spaced oscillation frequencies superposed on each other (Gleick, 2008).

This oscillation creates the possibility that a sophisticated arbitrager may analyze the frequency spectrum of market fluctuations to find an oscillation pattern large enough (because it represents a large enough group of investors with sufficiently similar time lags making a significant enough volume of trades) and persistent enough to provide an opportunity for above-market returns using a contrarian strategy (Klock & Bacon, 2014). Of course, applying the contrarian strategy tends to damp the oscillation. If enough investors apply it, the oscillation disappears, restoring weak-form efficiency.

## Proposed Theoretical Framework

The framework suggested in this paper consists of an equilibrium market value set by economic factors and perturbed by Shiller's (2003) feedback mechanism, which operates independently of economic forces. The equilibrium value follows Hayek's (1945) price-setting mechanism operating under Fama's (1970) strong EMH conditions (Klock, & Bacon, 2014). The feedback-mechanism model is a chaotic assemblage of phase-shift oscillators representing innumerable large and small investors requiring different amounts of time (delays) to make investment decisions.

In this paper, an *investor* is an individual or group of individuals buying or selling equities in an open market based on their analysis of all relevant public information (Fama, 1970). This analysis follows a cyclical decision-making process shown in Figure 1. The cycle starts at the top with observations of whatever external or internal data the investor considers relevant to the buy/hold/sell decision. The second step consists of qualitative and quantitative analysis of the gathered data. The third step draws a conclusion from that analysis, which results in a buy, hold, or sell decision. The final step executes that decision (usually through a broker) by finding a willing counterparty, negotiating a strike price, and completing a transfer of title to the asset. Finally, the cycle is closed by returning to the observation step to prepare for the next transaction. This process can be modeled as a human-in-the-loop (HIL) control system (Brogan, 1985).

**Figure 1**

*The cyclical investor decision-making process requires a finite but unspecified amount of time.*

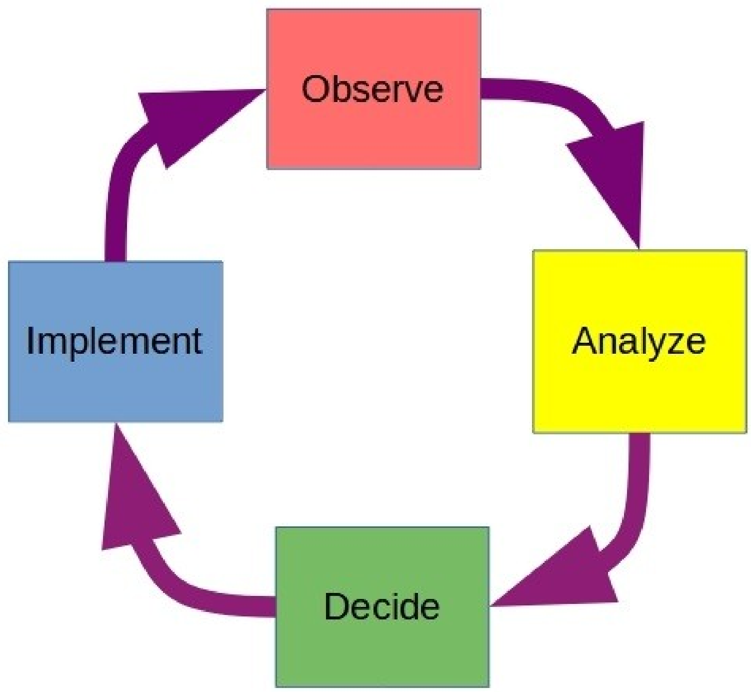
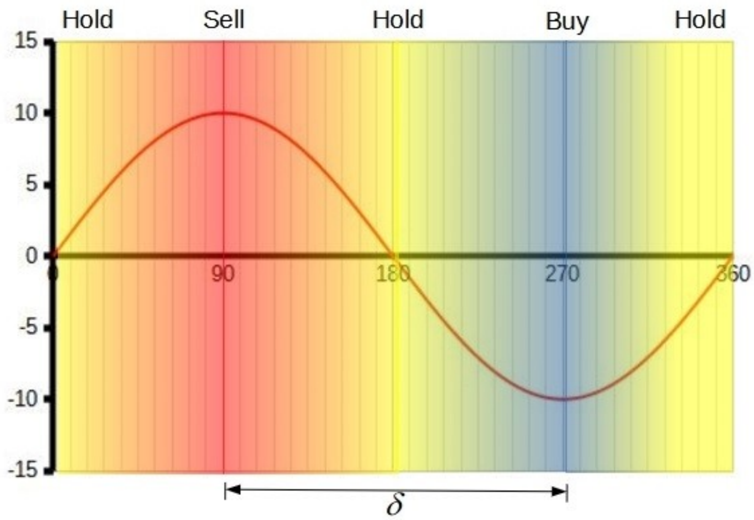



Figure 2 shows the basic buy low, sell high investment strategy (Watari et al., 2022). The strategy assumes that equities prices generally move up and down (depicted in Figure 2 as a sine wave). The strategy motivates investors to decide to sell when the market price is high and to buy when it is low, and to hold during transition periods.

What is significant for the suggested framework is that the investment cycle in Figure 1 takes time. The amount of time to complete the cycle is highly variable. This cycle time inserts a delay ($\delta$) into the decision process between the initial observation step and the final execution step characterizing the behavior of each investor. This delay makes the HIL feedback control system oscillate at a frequency equal to $1/2\delta$ (Efremov et al., 1996).

**Figure 2**

*The basic buy-low, sell-high investment strategy.*




The proposed framework models an equities market as an assemblage of a large number of investors, each modeled as a phase-shift HIL oscillator with a characteristic delay time and frequency. Signals from multiple investors having the same (or closely similar) delay times combine to increase the overall signal amplitude at that frequency.

## Methodology

Perturbation methods divide a problem into an easily solvable equilibrium component and a perturbation component as shown in equation 1 (Weinberg, 2021):

$$U(t)=U_0(t)+U'(t), \tag{1}$$

where $U_0(t)$ is the easily solvable equilibrium component and $U'(t)$ is a perturbing component. The framework assumes that the perturbation is a Taylor series of the form

$$U'(t)=\sum_{n=1}\varepsilon^n \varphi_n(t). \tag{2}$$

where $\varepsilon$ is an amplitude coefficient, and the $\varphi_n$ are functions to be determined by governing equations (Strogatz, 2018). Generally in perturbation analysis it is hoped that the first one or two terms in the series contain all of the useful information about the system being modeled, but this is not a requirement. The framework allows an unspecified number of components.

For the framework presented here, the problem being modeled is behavior of equity-market prices, specifically the Dow Jones Industrial Average (DJIA), which represents a portfolio of thirty prominent companies listed on U.S. stock exchanges (Turcaș et al., 2022). This study used the DJIA because it has a long, unbroken record reaching back to the late 19th century. Specifically, the study used data from the relatively quiet period from the end of the Great Depression (29 April 1932) to just before the disturbance caused by the COVID-19 epidemic (31 December 2019) (Amadeo, 2021).

**Equilibrium Component**

Strong-form EMH posits that equities market prices reflect current net-present values of future earnings of the companies they represent. The framework assumes that the equilibrium component of the market-price function has the same form as the time evolution of the value of the underlying asset. In the case of the DJIA, the underlying asset is the net present value of the companies in the index, which, in turn, represent the value of the underlying U.S. economy.

***Real GDP***

The framework assumes that economic development as measured by gross domestic product (GDP) generally follows a growth function of the form

$$U(t) = \sum_n \alpha_n \exp(\beta_n t) \tag{3}$$

where $U(t)$ is the GDP at time $t$, $\alpha_n$ is a coefficient for the growth term $n$, and $\beta_n$ is a coefficient to normalize time. The general form is thus a polynomial whose terms are exponentials. The zero-order term (n = 0) provides no growth (i.e., a constant GDP). The first order (n=1) term provides exponential growth with a constant growth rate. If $\beta_n$ for a given term is negative, the contribution for that term is negative and the GDP shrinks.

This provided the first hypothesis for the study:

- **H1** U.S. GDP growth took the form of an increasing exponential function during the period of interest.

  The null form of this hypothesis is:

- **H1$_0$** U.S. GDP did not grow at a constant rate during the period of interest.

The test for hypothesis H1 is to fit an increasing exponential function of the form shown in equation 4 to the DJIA longitudinal record:

$$U_0(t) = \alpha \exp(\beta t), \tag{4}$$

where $\alpha$ is the value of the function at the starting time $t = 0$, and $\beta$ is the growth rate normalized for the observation interval.

***EMH and Economic Development Trend***

As described above, EMH suggests that the DJIA time series should have the same form as the GDP time series. This provided a second hypothesis:

- **H2** The historical DJIA equilibrium price behavior showed the same functional form as time-series behavior of the U.S. GDP.

  It's null version is:

- **H2$_0$** The DJIA equilibrium price behavior did not match the functional form of the GDP time-series.

## Excess Volatility

Prior researchers have established that DJIA closing prices present frequent and significant deviations from the equilibrium prices suggested by the EMH model (Bouchard, 2021). The framework posits that these deviations act like signals from oscillators representing actions of a large, chaotic assemblage of investors operating with different reaction-time delays. This provided the third hypothesis:

- **H3** DJIA closing-price excess volatility has the functional form of a large chaotic assemblage of oscillators operating over a wide range of frequencies.

  Its null version is:

- **$H3_0$** DJIA closing-price excess volatility departs significantly from the functional form of signals from a large chaotic assemblage of oscillators.

Substituting an excess-volatility variable *V*(*t*) for *U'*(*t*) in equation 1 and solving or it shows the perturbation term to be equal to the DJIA data minus the exponential growth model:

$$V(t) = D(t) - U_0(t), \tag{5}$$

where *V*(*t*) is the excess volatility and *D*(*t*) is the historical DJIA closing price at time *t*.

**Discrete Fourier Transform**

The framework predicts that the excess volatility in equation 5 will have the form of a superposition of sinusoidal oscillations whose frequencies will depend on the time delays ($\delta_m$) individual investors require to make investment decisions and whose amplitudes will depend on the number and appetite of investors at each $\delta_\mathrm{m}$ value:

$$V(t) = \sum_m A_m \exp(2\pi i f_m t) \tag{6}$$

where $A_m$ is the amplitude of the oscillation at the frequency $f_m = 1/2\delta_m$.

The appropriate technique to analyze signals of this type is the discrete Fourier transform, which spreads a time-series of observations into a spectrum representing the complex amplitude of signals within evenly spaced frequency intervals (Stone, 2021). Using Pythagoras' theorem at each *m*th frequency yields the power carried by that oscillation as $A_m^2$. This power spectrum can yield insights into the mechanisms driving the process generating the signal.

A general property of chaotic oscillations is that their power spectra typically follow Zipf's Law (Kohyama, 1984). This ensures the fractal self-similarity property of a chaotic time series, and provides a fourth hypothesis to confirm H3:

- **H4** The power spectrum of excess-volatility oscillations follows Zipf's Law.

  The null version of this hypothesis is:

- **H4$_0$** The power spectrum of excess volatility oscillations does not follow Zipf's Law.

The appropriate test for confirming Zipf's Law is to perform linear regression analysis on a log-log scatter plot of the power spectrum (Strogatz, 2019).

## Results

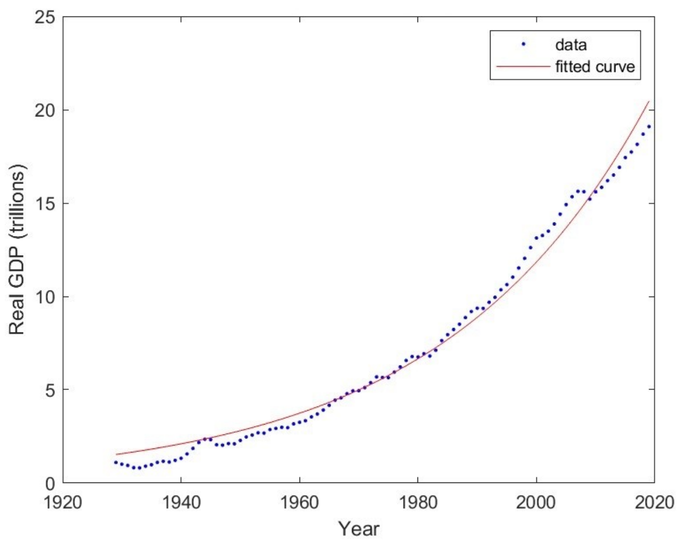
The analysis for this paper was carried out using two scripts written using the scripting language Matlab. The first script tested H1 by fitting a single exponential growth function to historical annual U.S. GDP data from 1932 (after the market recovered from the Great Depression) through 2019 (before the effects of the COVID-19 pandemic were felt in the U.S.) (Amadeo, 2021). The second script tested H2 by performing similar analysis on historical daily closing DJIA data from 29 August 1932 through 31 December 2019. The second script continued on to test H3 by analyzing the residuals of the exponential regression of DJIA data using Fourier transform methods. Finally, the second script tested H4 by performing linear regression on $log(A^2)$ versus $log(f)$.

### GDP Model

The test for hypothesis H1 consisted of fitting a one-term exponential regression function to historical GDP data. The data shown in Figure 3 consist of an exponential growth model fitted to annual real U.S. GDP data from 1932 through 2019 (Amadeo, 2021). Except for minor deviations, the GDP data match the first order exponential model ($\alpha$ = 1.195e-24, $\beta$ = 0.02878) quite well (Pearson's $R^2$ = 0.978). This confirms the hypothesis, so the framework may use an increasing exponential growth model with $n$ = 1 as the EMH-equilibrium component.

**Figure 3**

*Exponential growth function fitted to historical GDP data.*



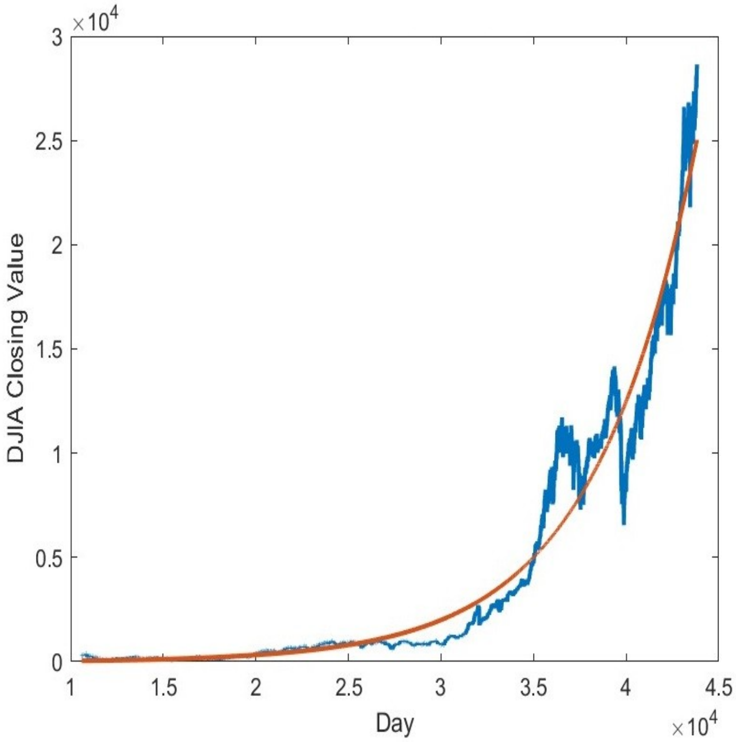

*NOTE*: From data compiled by the Bureau of Economic Statistics.

## Matching GDP and DJIA Trends

Figure 4 shows an increasing exponential model (solid line) fitted to historic DJIA closing values. The model parameters are $\alpha$ = 0.258 (95% boundaries being 0.2165 and 0.2994), $\beta$ = 0.04876 (95% boundaries being 0.04682 and 0.0507) and Pearson's $R^2$ = 0.9855. This result confirms hypothesis H2: DJIA equilibrium closing prices follow the GDP exponential growth curve.

**Figure 4**

*Exponential growth function fitted to DJIA closing values.*




**Residuals Power Spectrum**

Figure 5 shows normalized residuals from subtracting the equilibrium price model from the actual data per equation 5, and normalizing by dividing the difference by the equilibrium price model evaluated at the same time. Note that the time-series shows small-amplitude oscillations at high frequencies superposed on larger-amplitude oscillations at lower frequencies (longer periods). This is characteristic of time series having a Zipf's Law power spectrum (Ectors et al., 2019). These normalized residuals have an RMS value (standard deviation) of 0.3325, or approximately one third of the concommitant equilibrium market price.

**Figure 5**

*Normalized residuals from fitting the equilibrium model to historical DJIA data.*

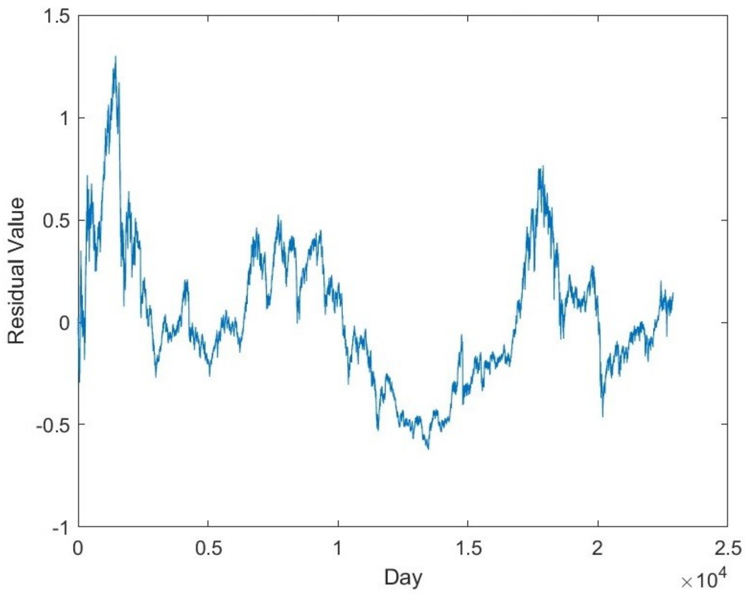


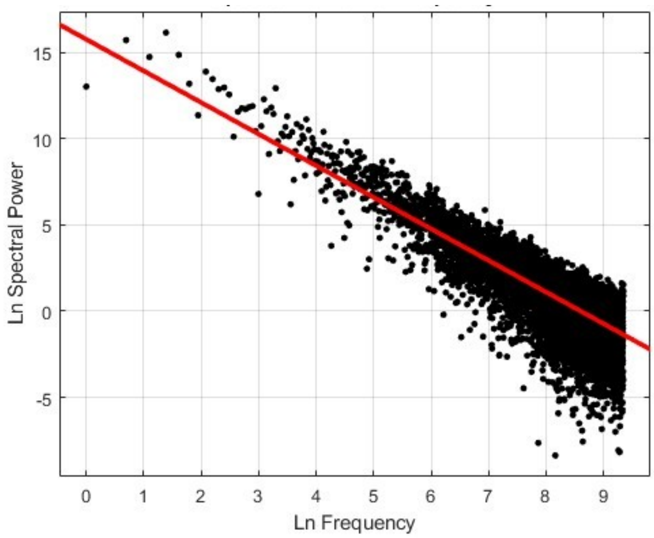

Figure 6 shows the results of discrete Fourier analysis of these residuals (Stone, 2021). Oscillation power levels range over 11 orders of magnitude and frequencies range over four orders of magnitude, necessitating a scatter plot on log-log axes to display the result. Figure 6 also includes a regression line fitted to the power spectrum showing Zipf's Law (inverse exponential) behavior and confirming H3 and H4. The regression model has a slope of -1.832 (95% confidence limits -1.855 to -1.808) and an intercept of 15.76 (95% confidence limits of 15.56 and 15.96) with $R^2 = 0.6684$. Taking the logarithm of both independent and dependent variables gives the model equation 7:

$$P(f) = 6.99E6 f^{-1.832}, \tag{7}$$

where $P(f)$ is the spectral power. This Zipf's Law behavior is a characteristic of chaotic fractal time series signals that exhibit self-similarity with a similarity dimension equal to the Zipf's Law exponent for the system (Strogatz, 2018).

**Figure 6**

*Residuals power spectrum shows Zipf's Law behavior.*




## Conclusion

This paper presented a framework for understanding the market-price activity of goods and services in public markets that explains the appearance of excess volatility invariably observed in those markets. The framework posited that prices are the sum of an equilibrium price reflecting the present value of the underlying good(s) or service(s) modified by a perturbation term dependent on non-economic factors reflecting negotiations between buyers and sellers. This framework led to four alternate hypotheses:

**H1** U.S. GDP growth took the form of an increasing exponential function during the period of interest.

**H2** The historical DJIA equilibrium price behavior showed the same functional form as time-series behavior of the U.S. GDP.

**H3** DJIA closing-price excess volatility has the functional form of a large chaotic assemblage of oscillators operating over a wide range of frequencies.

**H4** The power spectrum of excess-volatility oscillations follows Zipf's Law.

The paper tested these hypotheses against public-record historical data for DJIA closing prices and real GDP for the United States amassed during the period between the end of the Great Depression and the start of the COVID-19 pandemic in the United States. This period was chosen because it was relatively free from major financial disruptions, and so provided a benign test environment.

**H1 and H2: U.S. GDP Growth and Expected Equilibrium Values**

Results of comparing real GDP data to framework predictions found good agreement with hypothesis H1 for the period. The GDP matched an increasing exponential regression model. Furthermore, residuals of comparing the DJIA closing values to a similar increasing-exponential regression model (ie., excess volatility) showed homoscedasticity.

The framework posited that negotiation between buyers and sellers can be modeled as a control system seeking the equilibrium price of the underlying asset's value through consensus. As such negotiations always involve a time delay between an intention to transfer title to the underlying asset and consummating the transaction, in active markets the control system can (and usually does) oscillate at a frequency whose period equals twice the negotiation delay. In large, active markets with many participants the framework suggests that the perturbation term can be modeled as a chaotic assemblage of oscillators representing investors operating with different time delays. The resulting power spectrum is expected to follow Zipf's Law.

The study began by using U.S. GDP growth during the period of interest as a proxy for the equilibrium value of DJIA assets. A single-term exponential growth model was fitted to historical GDP data and found to explain 97.8% of the data during the period. Based on that result, the study fitted a similar increasing exponential growth model to the DJIA daily-closing values in the period, and found close agreement (Pearson's $R^2 = 0.9855$), which confirmed the equilibrium-price term in the framework.

The study then used Fourier analysis to compute the power spectrum of normalized residuals from fitting the exponential-growth model to DJIA data. The normalized residuals showed a 0.3325 standard deviation, meaning that the deviations generally amounted to one third of the estimated equilibrium value at each point in the time series. The power spectrum agreed with a Zipf's Law model, exhibiting a self-similar fractal form with a similarity dimension of -1.822 (Strogatz, 2018).

The study reported here had several limitations. Most obviously, it included only analysis of daily closing DJIA data. There are multiple other indices (e.g., S&P 500, NASDAQ, etc.) to which the framework should apply equally well. Additional research is needed to test the framework against these indices and against similar non-equities markets.

The methodology used in this study assumed many tens of thousands of investors in the market. One might ask how the results would differ when the market analyzed has fewer participants.

The fact that the data set included only daily closing prices limited the discrete Fourier analysis to frequencies below one-half cycle per day (Stone, 2021). Similarly, the use of a time series 87 years long made the analysis insensitive to frequency components lower than 1.15 cycles per century. The analysis could be extended to higher frequencies by incorporating intraday data. Extending the analysis to lower frequencies would require examining a longer time period.

Using longer time series would require a methodology more able to deal with significant perturbations at short time scales, such as the COVID-19 pandemic, which may last only a few years, or to decade-scale disruptions like the Great Depression. Wavelet analysis, which specifically searches for time-limited disturbances in time-series data, could prove helpful for this research (Vahid et al., 2022). Ultimately, the research reported here should be viewed as an important first step in understanding deviations from EMH.

**References**

Amadeo, K. (2021). U.S. GDP by year, compared to recessions and events: The strange ups and downs of the U.S. economy since 1929. *The Balance*. https://www.thebalance.com/us-gdp-by-year-3305543

Baron, M., Brogaard, J., Hagströmer, B., & Kirilenko, A. (2019). Risk and return in high-frequency trading. *Journal of Financial & Quantitative Analysis*, *54*(3), 993–1024.

Bhattacharya, S. N., Bhattacharya, M., & Roychoudhury, B. (2017). Behavior of the foreign exchange rates of BRICs: Is it chaotic? *Journal of Prediction Markets*, *11*(2), 1–18.

Bouchard, J. (2021). *Hypothesis (EMH) 1 efficient market hypothesis (EMH)*. Researchgate. https://www.researchgate.net/publication/356541414_HYPOTHESIS_EMH_1_Efficient_Market_Hypothesis_EMH

Brogan, W. L. (1985). *Modern control theory* (2nd edition). Prentice-Hall.

Chauhan, Y., Chaturvedula, C., & Iyer, V. (2014). Insider trading, market efficiency, and regulation. A literature review. *Review of Finance & Banking*, *6*(1), 7–14.

Ectors, W., Kochan, B., Janssens, D., Bellemans, T., & Wets, G. (2019). Exploratory analysis of Zipf's universal power law in activity schedules. *Transportation*, *46*(5), 1689-1712.

Efremov, A. V., Rodchenko, V. V., & Boris, S. (1996). *Investigation of Pilot Induced Oscillation Tendency and Prediction Criteria Development (No. SPC-94-4028)*. Moscow Inst of Aviation Technology (USSR).

Fama, E. F. (1970). Efficient capital markets: A review of theory and empirical work. *Journal of Finance*, *25*(2), 383-417.

Gleick, J. (2008). *Chaos: Making a new science*. New York, NY; Penguin Group.

Hayek, F. A. (1945). The Use of Knowledge in Society. *American Economic Review*, *35*(4), 519–530.

Inglada-Perez, L. (2020). A comprehensive framework for uncovering non-linearity and chaos in financial markets: Empirical evidence for four major stock market indices. *Entropy*, *22*(12), 1435.

Klock, S. A., & Bacon, F. W. (2014). The January effect: A test of market efficiency. *Journal of Business & Behavioral Sciences*, *26*(3), 32–42.

Kohyama, T., & Aizawa, Y. (1984). Theory of the intermittent chaos: 1/f spectrum and the Pareto-Zipf law. *Progress of Theoretical Physics*, *71*(5), 917–929.

Lorenz, E. N. (1963). Deterministic nonperiodic flow. *Journal of the Atmospheric Sciences*, *20*, 130-141.

Meiyanti, R., Sensuse, D. I., & Sucahyo, Y. G. (2021). The main component of smart tourism: A principal component analysis approach. *Webology*, *18*(2), 295-307.

Shiller, R. J. (2003). From efficient markets theory to behavioral finance. *The Journal of Economic Perspectives*, *17*(1), 83-104.

Stone, J. V. (2021). *The Fourier transform: A tutorial introduction*. Sebtel Press.

Strogatz, S. H. (2018). *Nonlinear dynamics and chaos.* Boca Raton, FL: CRC Press.

Turcaș, F., Florin, C. D., & Boiță, M. (2022). Econophysics techniques and their applications on the stock market. *Mathematics*, *10*(6), 860.

Vahid, G., Meena Gopikrishnan, G., Wall, T. A., Stott, M. A., Hawkins, A. R., & Schmidt, H. (2022). Fast custom wavelet analysis technique for single molecule detection and identification. *Nature Communications*, *13*(1). https://doi.org/10.1038/s41467-022-28703-z

Watari, H., Takayasu, H., & Takayasu, M. (2022). Analysis of Individual High-Frequency Traders' Buy–Sell Order Strategy Based on Multivariate Hawkes Process. Entropy, 24(2), 214.

Weinberg, S. (2021). *Foundations of modern physics.* Cambridge University Press.